Restoration of Traffic Flow Data Based on Tensor Decomposition

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Abstract

The collection of traffic data is the first step of urban traffic flow forecasting. It is very important to grasp the change rules of urban traffic flow, grasp the evolution characteristics of urban road traffic system, and formulate scientific traffic control measures. The sampling data of traffic detectors is usually a part of the missing high-dimensional form. The traffic data in this paper contains rich multi-mode features to interpolate the missing parts. Taking the local road network of Hefei as the research object, three tensor restoration methods were used to repair the traffic flow data under different loss rates respectively, and the accuracy of data repair under the different missing rates of two cases of random loss and structural loss were discussed.

Keywords: Traffic flow; Tensor decomposition; Missing data; Data imputation.

INTRODUCTION

Traffic management and traffic guidance systems require accurate traffic information. However, data noise, redundant repetition, and lack of information caused by equipment failures, communication failures, and environmental impacts cause many difficulties for subsequent applications[1]. By obtaining comprehensive and abundant traffic information, we can grasp the urban road traffic conditions and changing laws, and provide scientific basis for urban traffic planning and decision-making.

Traffic data repair methods can be roughly divided into three categories[2]: vector-based repair methods, matrix-based repair methods, and tensor-based repair methods. Signoretto M[3] proposes tensor to maximize the inherent structural characteristics of data. Ran B[4] proposes a tensor-based approach that considers the full temporal and spatial information of traffic flow to fuse traffic flow data from multiple inspection sites.WuYuankai [5] proposed a tensor filling theory based on matrix decomposition, combined with the dynamic tensor model of traffic flow and the tensor filling method based on matrix decomposition, applying the dynamic tensor filling method step by step to apply short-term traffic flow prediction. According to the multi-mode characteristics of traffic data, this paper observes from different angles of day, week, space and time, the traffic data shows a strong multi-correlation in these

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[5[]] WU Yuankai . Short-term traffic prediction based on dynamic tensor completion[D].Beijing Institute of Technology,2015 modes. Use high-dimensional tensors for analysis. Based on the characteristics of traffic data, it can be constructed in the form of a tensor of road section×day×hour×flow.

The structure of this paper is as follows: Section 2 describes the principle of data restoration based on tensor decomposition. Section 3 tests the algorithm based on the traffic flow data set to verify the effectiveness of the algorithm. Section 4 summarizes this article .

The Principle of Data Restoration Based on Tensor Decomposition

In this paper, an optimization method based on tensor decomposition is used in tensor filling theory. The method is usually divided into CP (CANDECOMP/PARAFAC) decomposition and Tucker decomposition. The models are generally shown as follows:

$$\min_A \frac{1}{2}\left\|\Omega * (A-y)\right\|_F^2 \tag{1}$$

Among them, Ω is a marked tensor, where the data is lost is 0 and the rest is 1.

CP decomposition model

The most important problem of the CP decomposition model is to set the rank of the tensor, but in general, the tensor is contaminated by noise and the calculation is always error. Therefore, the model is designed to solve equation (2):

$$\min_X \left\|X - \hat{X}\right\| \tag{2}$$

Among them, $\hat{X} = \sum_{r=1}^{R} \lambda_r \alpha_r \circ b_r \circ c_r \circ d_r = \left\|\lambda, A, B, C, D\right\|$. Equation (2) can be understood as: find the most appropriate λ,A,B,C and D, so that the norm of the error between the original tensor and the decomposition model is as small as possible. The better algorithm to solve this problem is the ALS algorithm. As shown in Table 1.

Table 1 CP-ALS Algorithm

Algorithm 1 *ALS algorithm to compute a CP decomposition with R components for an Nth-order tensor $\mathcal{X}$ of size $I_1 \times I_2 \times \cdots \times I_N$*

initialize $A^{(n)} \in \mathbb{R}^{I_n \times R}$ for $n = 1, \cdots N$
repeat
  for $k = 1$ **to** N **do**
    $V \leftarrow V = A^{(1)T}(A^{(1)} \odot \cdots A^{(n-1)} \odot A^{(n+1)} \cdots \odot A^{(N)T} A^{(N)})$
    $A^{(n)} = X^n (A^{(N)} \odot \cdots A^{(n+1)} \odot A^{(n-1)} \cdots \odot A^1) V^\dagger$
    normalize columns of $A^{(n)}$(storing norms as λ)
  end for
until fit ceasese to improve or maximum iterations exhausted
return $\lambda, A^{(1)}, A^{(2), \cdots A^{(N)}}$

The CP-WOPT algorithm[6] can fill high-dimensional data, give a tensor *X* with missing partial data, and define a weight tensor *W* with missing information as shown in formula (3):

$$w_{i_1 i_2 \dots i_N} = \begin{cases} 1 & \text{If } x_{i_1 i_2 \dots i_N} \text{ is konwn} \\ 0 & \text{If } x_{i_1 i_2 \dots i_N} \text{ is missing} \end{cases} \tag{3}$$

The objective function is:

[6] Acar E, Dunlavy D M, Kolda T G, et al. Scalable tensor factorizations for incomplete data [J]. Chemometrics & Intelligent Laboratory Systems, 2010, 106(1):41-56.

$$f_W(A^{(1)},A^{(2)},...A^{(N)})=\frac{1}{2}\left\|(W*X)-W*\left[A^{(1)},A^{(2)},...A^{(N)}\right]\right\|_W^2 \Leftrightarrow \frac{1}{2}\left\|Y-Z\right\|^2 \quad (4)$$

Among them, $Y=W*X;Z=W*\left[A^{(1)},A^{(2)},...A^{(N)}\right]$, The problem is converted to $A^{(1)},A^{(2)},...A^{(N)}$ so that the objective function value *f* is minimized.

Tucker decomposition model

Similar to the idea of the CP-ALS algorithm, in order to solve equation (4), an alternating least-squares method is used, and the objective function and the core tensor calculation formula are:

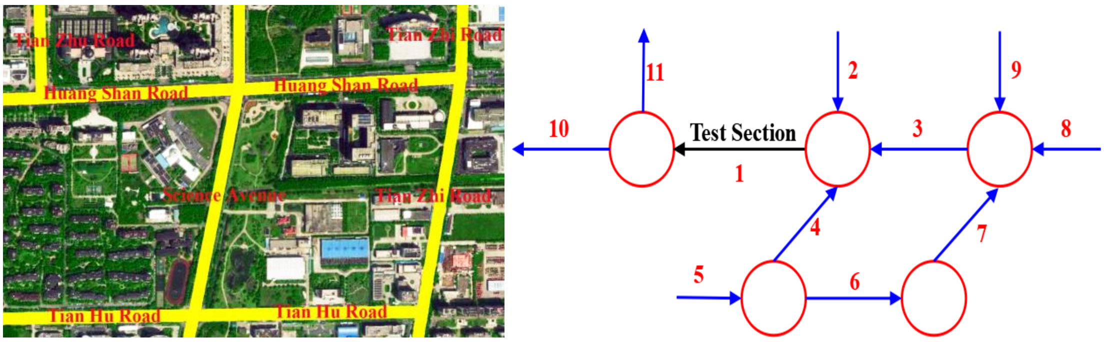
$$f_W(A^{(1)},A^{(2)},...A^{(N)})=\frac{1}{2}\left\|(W*X)-W*\left[\Phi\square\ A^{(1)},A^{(2)},...A^{(N)}\right]\right\|_W^2 \quad (5)$$

$$\Phi = A\times_1 A^{(1)T}\times_2 A^{(2)T}\times\square\times_N A^{(N)T} \quad (6)$$

The TUCKER-ALS algorithm uses the idea of least squares to initialize all matrices except one of the factor matrices, through repeated iterations until the function value converges or the maximum number of iterations is reached.

CASE STUDY

In this paper, the partial road network oft Huangshan road in Shushan District of Hefei is selected as an object of verification, and an example is analyzed. The actual road is shown in Figure 1. Among them, the section to be tested (No.1) is a section of Huangshan routing eastward to the west, located between the science Avenue and Tianzhu Road, and the length of the road is about 430 meters. The upstream section of the section to be tested includes 2-9 of the road section and 10-11 of the downstream section.



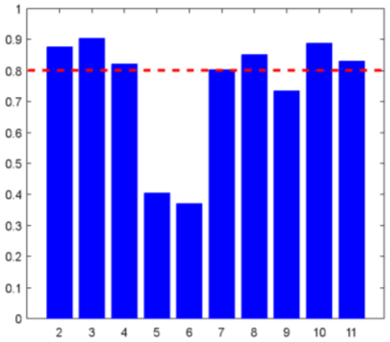
(a) Actual local road network diagram structure

(b) Schematic diagram of local road network

Fig 1 Network diagram in a case

Correlation Analysis

Use the data of 2016.07.04--2016.07.19 as an example, and use the statistical traffic flow data every 5 minutes as an example. The upstream and downstream sections have different effects on the road sections to be measured. In order to find the upstream and downstream sections that have strong correlation with the sections to be measured, Person correlation analysis is needed on the traffic data of the sections. The results are shown in Figure 2.



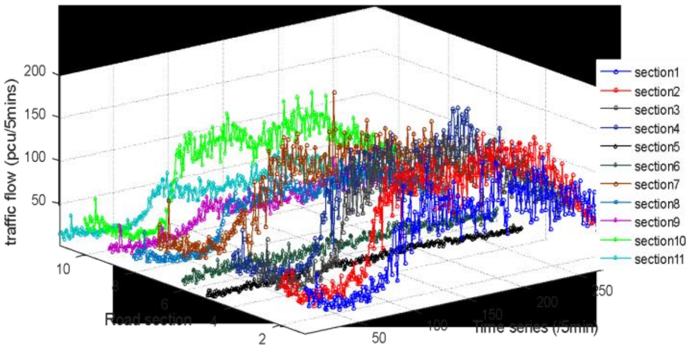


Fig 2 Relevance of the road section **Fig 3** Traffic Trends for Different Sections on July 4, 2016

As can be seen from Fig 2, except for the sections 5 and 6, and the section 9, the correlations between the other sections and the section to be measured are all greater than 0.8. Therefore, traffic data of 8 sections of the road to be measured and the remaining 7 sections are constructed to construct a history database. The historical database contains the data of these eight sections 2016.07.04--2016.07.19 for a total of 16 days, and the test set data is the traffic volume of the test sections 07.18 and 07.19.

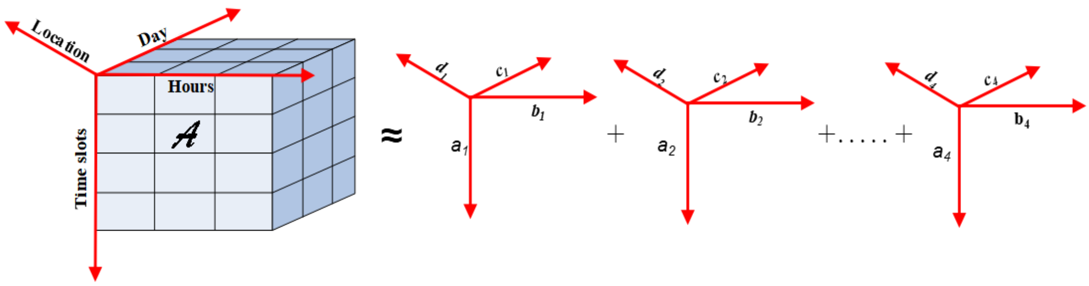
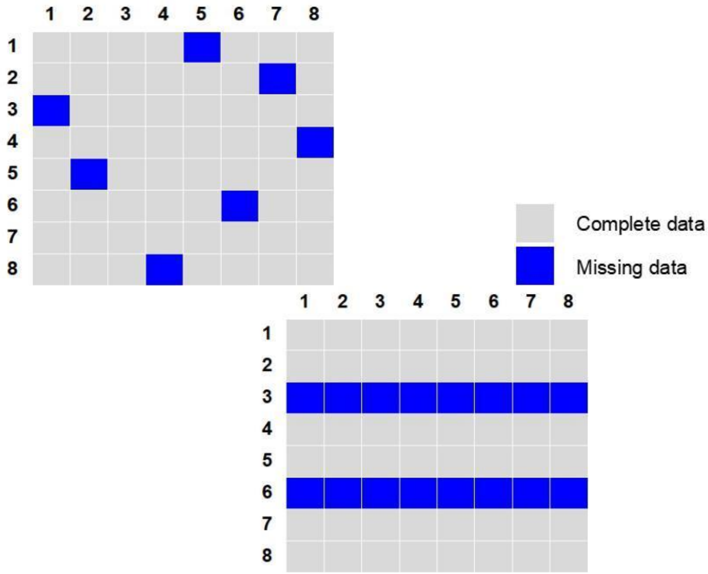


Fig 4 The CP decomposition model of 4 dimensional tensor

According to the acquired data, the historical database is constructed as a four-dimensional tensor with a size of 8×16×24×12. The space-time related information such as space mode and day mode can be used simultaneously. The four dimensions respectively represent 8 correlations. The road section, 16 days, 24 hours, 12 flows per hour, as shown in Fig 4. Figure 3 shows the trend of traffic changes on the same day in different road sections. Although the traffic volume of different road sections is different, the overall trend of change has a strong similarity, which is related to the spatial shift of traffic flow on each road section.

Repair result

Based on the given local road network data, three data restoration experiments on CP-ALS, CP-WOPT and TUCKER-ALS based on tensor decomposition-based missing data were performed. Data missing types are divided into two kinds: random missing and structural missing. Randomly missing data randomly selected 10%, 20%, ..., 90% of the 16-day historical data of 8 sections. Structural Missing The same amount of data is extracted from the same historical data Hypothesis is missing, but in the extraction method, it assumes that the missing data occurs in different sections of the same time period. Figure 5 shows two kinds of missing data: missing data and missing structural data[7].



(a) Missing at random
(b) Structural loss

Fig 5 Missing data type

Data repair results under missing random

In order to display the repair effect of cp_wopt more intuitively, Figure 6 shows the comparison effect of the missing data and repaired data of the road segment on July 18 when the data is missing 90%, and the comparison effect between the real data and the repaired data. From Fig. 7, it can be seen that although the known data information is few, the data recovered by cp_wopt can completely reflect the fluctuation of the traffic flow.

Fig 6 Tensor repair error in random loss:(a) MSE(b)Randomly missing 90% data repair results:Comparison of missing data and repair data; (c) Randomly missing 90% data repair results:Comparison between real data and repair data

Data repair results under structural loss

Figure 7(a) shows the errors of the three data repair algorithms for cp_als, cp_wopt, and tucker_als in the absence of structural errors, taking the average of 15 experiments. From figure 7(a), we can see that in the absence of structurality, the effect of the three methods for repairing data is similar to that in the case of random loss. The error of cp_als tucker_als is large, but the error of cp_wopt is small, and the error does not follow. Missing rate increases.

7[] Li Meng.Urban short-term traffic flow prediction under incomplete information[D] Central South University. 2016.

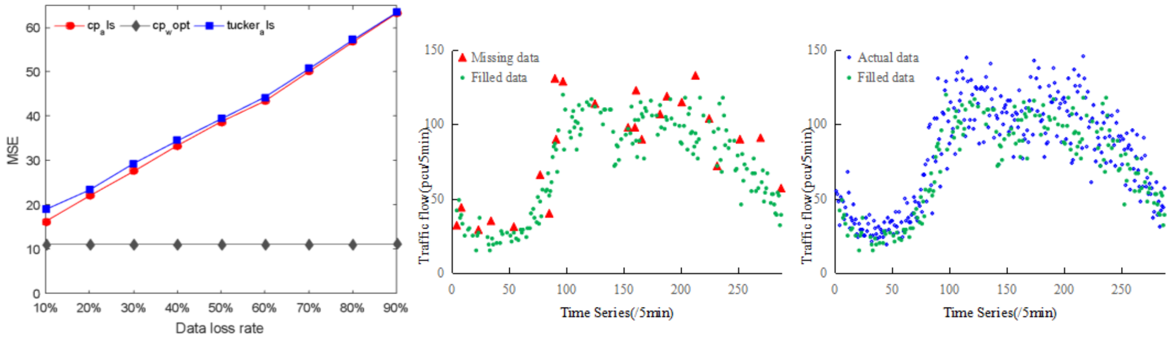


Fig 7 Tensor repair error in structural loss:(a) MSE; (b)Structural missing 90% data repair results:Comparison of missing data and repair data; (c) Structural missing 90% data repair results:Comparison between real data and repair data.

Figure 7(b) shows the effect of missing data and repair data on the road segment of July 18 when the data missing rate is set to 90% in the absence of structurality. Figure 7(c) shows the contrast between real data and repair data. It can be seen that under 90% structural loss, the data repaired by the cp_wopt algorithm can completely reflect the fluctuation trend of traffic flow, but it cannot predict well the data with large deviation.

CONCLUSION

Traffic flow data has significant multi-mode characteristics. In this paper, tensor analysis is introduced into traffic flow data analysis, and traffic flow data is constructed as a 4th order tensor model. Based on this, the partial road network in Hefei is taken as the research object, and the tensor repair method is used for different missing rates. The traffic flow data under the repair was used to investigate the accuracy of data restoration under different missing rates in the absence of randomness and loss of structure. Example results show that in the absence of random and structural loss, CP-ALS and TUCKER-ALS The repair errors of the ALS algorithm gradually increase with the increase of the missing rate, while the repair error of the CP-WOPT is basically stable at the same level, and the error is smaller than the former two.

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Reference